Indian Statistical Institute, Bangalore Centre
B.Math (Hons) III Year, Second Semester
Mid-Sem Examination
Comb. and Graph Theory
Time: 3 Hours   February 22, 2012   Instructor: N.S.N.Sastry
Total Mark is : 100

Note: Answer all questions. Your answers should be complete and clearly written.

1. Determine for which of the following parameters does a $t - (v, k, \lambda)$ design exists. If it exists give a construction. If it does not, justify your answer.

   (i) $2 - (57, 7, 1)$ (ii) $3 - (33, 7, 1)$ [10]

2. Let $V = \mathbb{F}_q^4$ and f be the bilinear form on V defined by

$$f((x_1, x_2, x_3, x_4), (y_1, y_2, y_3, y_4)) = (x_1 y_2 - x_2 y_1) + (x_3 y_4 - x_4 y_3).$$

   Show that:

   (a) f is a non-degenerate bilinear form;

   (b) The number of isotropic vector subspaces of dimension 2 is $(q + 1)(q^2 + 1)$.

   [10+12]

3. Let U be a subspace of $\mathbb{F}_q^n$ of dimension k and $1 \leq k \leq l \leq [\frac{n}{2}]$. Find the number of subspaces of $\mathbb{F}_q^n$ of dimension l not containing U.   [10]

4. Determine the largest size of a binary 3-error correcting code of length 32.   [8]

5. Define the rank of a group acting transitively on a set X. Determine the rank of the action of $GL(n, \mathbb{F}_q)$ on the set X of $(n-2)$ dimensional subspaces of $\mathbb{F}_q^n$.   [4+8]

6. Give one example for each of the following situations:

   (a) a 1-transitive, but not a 2-transitive action of a group on a set;

   (b) a 2-transitive, but not 3-transitive action of a group on a set.

   [4+4]

7. Define a projective plane of order n and an affine plane of order n. Show that a projective plane of order n exists if, and only if, an affine plane of order n exists.   [10]

8. (a) Show that the complement of each block in a $5 - (12, 6, 1)$ design is again a block.

(b) Show that a $4 - (11, 5, 1)$ design has a unique one-point extension.

[12+8]